

# Numerical Studies of Acoustic Propagation in Shallow Water

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## LONG-TERM GOALS

To develop “exact” numerical methods and visualization techniques for the study of propagation of acoustic energy in shallow water in the time domain. Exact in this contexts means the methods place no restrictions on the underlying physics of the environment.

## OBJECTIVES

To develop new, and enhance existing, numerical methods; to establish the accuracy, robustness, flexibility, and tractability of these methods; and to apply these methods to meaningful practical problems. To create a robust software suite for the application of the methods and to develop techniques for visualizing the propagation of scalar and vector fields in complex environments.

## APPROACH

We have focused our attention on the development and use of finite-difference time-domain (FDTD) methods. These time-domain techniques, which are flexible, robust, and generally simple to implement, have been used extensively in the past by the electromagnetics community to solve a wide range of problems. We have established the accuracy of our proposed acoustic FDTD techniques by comparing numeric results to exact solutions for canonical problems or to results obtained using other numerical methods. The FDTD method is computationally intensive and, hence, to increase the ease with which FDTD can be applied to large problems, we are developing code which incorporates the Parallel Virtual Machine paradigm to permit the solution of a problem using a cluster of workstations.

## WORK COMPLETED

Fundamental aspects of material boundaries in FDTD models were studied and quantified; inherent properties and artifacts of the FDTD grid were rigorously explained; FDTD algorithms were developed; the corresponding computer programs were written; and information was disseminated via journal publications, conference presentations, and the Web.

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## RESULTS

The FDTD method is obtained by discretizing the differential equations that govern the underlying system. Using a Cartesian grid, the method provides an exceedingly simple way in which to express future fields (i.e., unknown fields) in terms of past fields (known fields). For propagation in a homogeneous region, the traditional FDTD method is accurate to second-order—that is, doubling the number of grid points per wavelength reduces inherent numerical errors by a factor of four. However, the behavior and accuracy of fields at material interfaces are much more complicated.

We previously derived exact expressions for the transmission and reflection coefficients for fields normally and obliquely incident on planar boundaries and showed that the behavior of the fields on one side of the boundary depends on the discretization on the other side [1,2]. However this work did not address the more challenging problem of the errors caused by the inherent “stair-step” approximation of material boundaries that are continuously varying. To date there has been no quantification of these errors for an arbitrary boundary. Nevertheless, for a given object one can codify these errors through measurements. We have used measurements to demonstrate the accuracy of the FDTD method when applied to various problems including randomly rough pressure-release surfaces [3,4,5], penetrable surfaces [6], and rigid boundaries [7,8].

In all our work modeling continuously varying boundaries we have developed and employed locally-conformal schemes which use modified versions of the FDTD updated equations for nodes adjacent to material interfaces but which use the standard equations elsewhere. In [5,7,8] we showed the improvements that such locally-conformal schemes can yield while introducing essentially no additional computational cost. To demonstrate the improvement that can be realized using a locally-conformal scheme, consider the rigid sphere shown in Fig. 1. The figure shows the discretized version of the sphere as it would appear in a Cartesian grid when discretized using a radius of eight cells. The sphere is ionized with a broad-spectrum pulsed plane wave and the scattered field is measured along the heavy line shown in Fig. 1. The time-domain results can be Fourier transformed to obtain the scattered field at any frequency for which the incident pulse has sufficient spectral content. The results in Fig. 2 show the scattered field corresponding to a wavelength which is modeled using 9.853 points per wavelength. The exact result is plotted together with those obtained using the FDTD scheme with a uniform, or staircase, grid (UFDTD) and using a locally-conformal scheme (CFDTD). Clearly the CFDTD approach provides a significant improvement over UFDTD. Ten points per wavelength is often the coarsest discretization employed in FDTD simulations. For the simulation geometry shown here the agreement between simulations and the exact result improves as the discretization is increased, but regardless of the discretization the locally-conformal CFDTD method provides results superior to those of UFDTD.

Despite the fact that we have used a locally-conformal scheme in the past to model penetrable objects [6], the improvement realized by using the locally conformal scheme was typically small. We have recently completed a rigorous and systematic comparison of various low-cost, locally-conformal schemes for penetrable objects. Various sound speeds, densities, object sizes, and discretizations were used. Unfortunately there was no clear “winner” in terms of the best scheme. However, when considered over all the test cases, the scheme which typically performed the best was arguably the simplest and closely akin to the staircasing technique: densities should be associated with pressure nodes, and velocity nodes

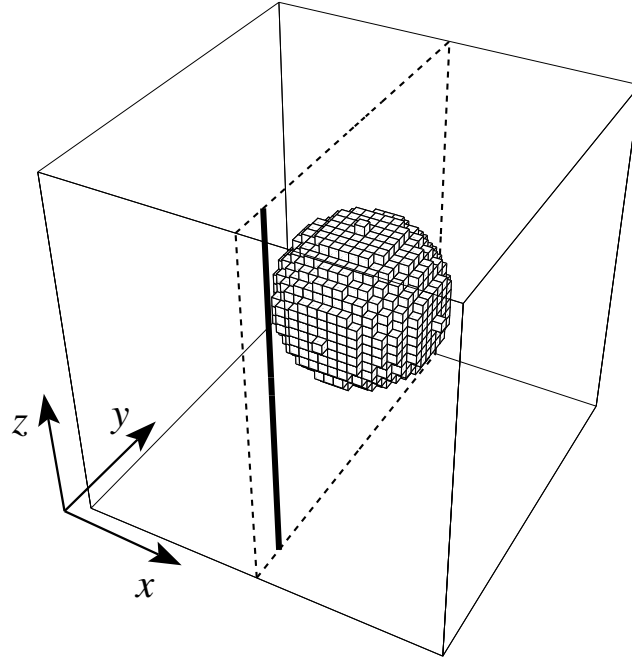


Figure 1: Representation of the 3D computational domain used to obtain the scattered pressure from a rigid sphere. The sphere is depicted in accordance with its stair-step representation. The incident plane wave travels in the  $+z$  direction and the scattered pressure is collected along the position of the heavy vertical line.

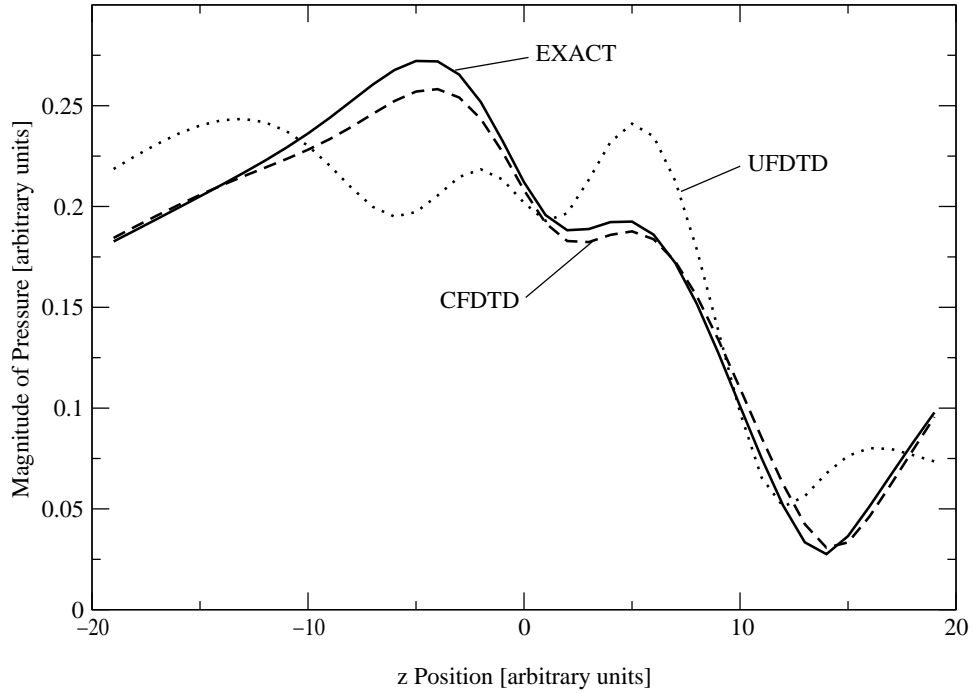


Figure 2: Scattered pressure at 9.853 points per wavelength. The independent variable is position in terms of cells where zero corresponds to the same  $z$  location as the center of the sphere.

whose neighboring pressure nodes span the material boundary should employ the average density in their update equations. This work will be submitted soon to an appropriate journal.

We continue to explore several new implementations of the FDTD method (proposed by others) which seek to minimize dispersive and anisotropic errors inherent in all 2- and 3-D FDTD schemes. We have developed comparisons that provide insight into the techniques that are not easily garnered from the publications in which they were originally presented. Some of this work will appear in *IEEE Transactions on Microwave Theory and Techniques* [9] and was presented at the 2002 URSI/Antennas and Propagation Symposium [10]. Notably, we demonstrated that many of the wavelet-based schemes, which have attracted some advocates, are not superior to an FDTD scheme that uses the same spatial stencil and the same “computational effort” (i.e., operations per a given temporal advancement of the fields).

Along these same lines, we have adapted a variation of the promising FDTD scheme proposed by Eric Forgy (*IEEE Transactions on Antennas and Propagation*, **50**(7):983–996, 2002). This algorithm suffers much less grid dispersion and anisotropy than more traditional FDTD formulations but still retains the local nature of the standard update equations. The acoustic implementation will be described in a future publication.

It has long been known that the Yee FDTD algorithm can provide exact solutions to one-dimensional problems when operated at the so-called magic time step. Here “exact” is taken to mean the field propagates without dispersion error or other numeric artifacts. Unfortunately there is no magic time step in higher dimensions. However we have recently developed a theoretical framework for multi-dimensional algorithms that have the same exact properties as the one-dimensional Yee algorithm when operated at the magic time step. The proposed technique uses vector operators which, instead of being defined at a point such as with the usual gradient, divergence, and curl operators, are defined over spheres. Due to their inherent symmetry, these spatial operators have the same properties in all directions. With a judicious choice of the temporal step size the temporal errors can cancel the spatial errors and the algorithm is exact. However, although the framework for the algorithm has been developed, no practical (i.e., computationally efficient) algorithm has yet been developed. Nevertheless proof-of-concept implementations of the algorithm (which are quite computationally expensive) have been used to demonstrate the validity of the technique and the improvements the algorithm can provide over other FDTD implementations. The algorithm also has interesting properties such as unconditional stability for an arbitrary temporal step size. We are continuing to explore this algorithm and will be publishing our findings in the near future.

We continue to maintain a Web site, [www.fdttd.org](http://www.fdttd.org), that seeks to list all archival publications related to the FDTD method. This site solicits input, in the form of comments posted about work appearing in the archival literature, from the entire community interested in the FDTD method (whether applied to acoustics, electromagnetics, or solid mechanics).

Our investigations of the discretized worlds of FDTD methods have led us to a better understanding of numeric artifacts associated with resonances and to ways of alleviating these artifacts. Part of this work was presented as an invited talk in a special session organized by Prof. Allen Taflov (one of the co-founders of the FDTD method) [11]. This work is further described in a paper submitted to *IEEE*

*Transactions on Antennas and Propagation* [8].

A recent publication by John Pendry (*Phys. Rev. Lett.*, **85**:3966, 2000) which described the use of backward-wave (BW) materials to make a “perfect lens” caught our attention. BW materials are dispersive materials whose direction of phase propagation is antiparallel to the direction of power flow. BW behavior can exist in both acoustic and electromagnetic systems. We were interested in modeling BW material using the FDTD method. Our initial results were not consistent with those one would expect from an initial inspection of the theory. Eventually we discovered that the dual, offset grids employed in the FDTD method (i.e., the dual pressure and velocity grids or the dual electric and magnetic field grids) can introduce significant numeric artifacts when modeling BW materials. The offset in the grids can introduce a boundary layer that has the material properties of neither the BW material nor the surrounding medium. Our investigations were presented in an invited talk at the 2002 URSI/Antennas and Propagation Symposium [13] and in a paper which was published in *Physical Review Letters B* [14].

## **IMPACT/APPLICATIONS**

Accurate and flexible numerical methods allow researchers to conduct any number of experiments without having to resort to actual field experiments, i.e., the experiment is conducted on the computer. Although numerical methods will never supplant field experiments, numerical methods (when used within their “region of validity”) do provide an extremely cost-effective means of conducting controlled experiments. Our work enables more accurate and more efficient numerical solutions to a wide range of problems in acoustics, electromagnetics, and continuum mechanics.

## **TRANSITIONS**

Much of the knowledge we have gained has been disseminated via publications and conference presentations. Additional material is available via the Web. Please refer to the Web site given in the header for copies of the PI’s publications or [www.fdttd.org](http://www.fdttd.org) for other material pertinent to the FDTD method.

## **RELATED PROJECTS**

This work is related to research in both high-frequency acoustics and long-range propagation. Numerical models, such as the FDTD method, can be used to predict the fields scattered from small objects under short-wavelength insonification or the propagation of long-wavelength signals over limited regions of the ocean. Additionally, this work is related to several other ONR-sponsored researchers including Shira Broschat, Eric Thorsos, Philip Marston, and Ralph Stephen.

## **REFERENCES**

- [1] J. B. Schneider and R. J. Kruhlak, “Plane Waves and Planar Boundaries in FDTD Simulations,” IEEE AP-S International Symposium and URSI Radio Science Meeting, Salt Lake City, UT, Jul. 2000.
- [2] J. B. Schneider and R. J. Kruhlak, “Dispersion of Homogeneous and Inhomogeneous Waves in the Yee Finite-Difference Time-Domain Grid,” *IEEE Trans. Microwave Theory and Techniques*, vol. 49, no. 2, pp. 280–287, 2001.

- [3] F. D. Hastings, J. B. Schneider, and S. L. Broschat, "A Monte Carlo FDTD Technique for Rough Surface Scattering," *IEEE Trans. Antennas Propagat.*, vol. 43, no. 11, pp. 1183–1191, 1995.
- [4] F. D. Hastings, J. B. Schneider, and S. L. Broschat, "A Finite-Difference Time-Domain Solution to Scattering from a Rough Pressure-Release Surface," *J. Acoust. Soc. Am.*, vol. 102, no. 6, pp. 3394–3400, 1997.
- [5] J. B. Schneider, C. L. Wagner, and R. J. Kruhlak, "Simple Conformal Methods for FDTD Modeling of Pressure-Release Surfaces," *J. Acoust. Soc. Am.*, vol. 104, no. 6, pp. 3219–3226, 1998.
- [6] F. D. Hastings, J. B. Schneider, S. L. Broschat, and E. I. Thorsos, "An FDTD Method for Analysis of Scattering from Rough Fluid-Fluid Interfaces," *IEEE Journal of Oceanic Engineering*, vol. 26, no. 1, pp. 94–101, 2001.
- [7] J. B. Schneider and J. G. Tolan, "A Locally Conformal Method for Modelling Rigid Boundaries in the FDTD Method," *J. Acoust. Soc. Am.*, vol. 108, no. 5, pt. 2, pp. 2563, Newport Beach, CA, Dec. 2000.
- [8] J. G. Tolan and J. B. Schneider, "Locally Conformal Method for Acoustic Finite-Difference Time-Domain Modeling of Rigid Surfaces," submitted to *J. Acoust. Soc. Am.*
- [9] K. L. Shlager and J. B. Schneider, "Comparison of the Dispersion Properties of Several Low-Dispersion Finite-Difference Time-Domain Algorithms," to appear in *IEEE Trans. Antennas Propagat.*, Dec., 2002.
- [10] K. J. Shlager and J. B. Schneider, "A 2-D Dispersion Analysis of the W-MRTD Method Using CDF Biorthogonal Wavelets," *IEEE AP-S International Symposium and URSI Radio Science Meeting*, vol. 3, pp. 244–247, San Antonio, TX, Jun. 2002.
- [11] C. L. Wagner and J. B. Schneider, "Using the Dispersion Relation to Understand Finite-Difference Time-Domain Worlds," *International Conference on Electromagnetics in Advanced Applications (ICEAA 01)*, Torino, Italy, pp. 375–378, Sep. 2001.
- [12] C. L. Wagner and J. B. Schneider, "On the Analysis of Resonators Using Finite-Difference Time-Domain Techniques," submitted to *IEEE Trans. Antennas Propagat.*
- [13] M. W. Feise, P. J. Bevelacqua, and J. B. Schneider, "Backward-Wave Meta-Materials for Perfect Lenses," *IEEE AP-S International Symposium and URSI Radio Science Meeting*, San Antonio, TX, Jun. 2002.
- [14] M. W. Feise, P. J. Bevelacqua, and J. B. Schneider, "Effects of Surface Waves on the Behavior of Perfect Lenses," *Physical Review B*, vol. 66, no. 3, 035113 (five pages), 2002.

## PUBLICATIONS

- [1] J. G. Tolan and J. B. Schneider, "Locally Conformal Method for Acoustic Finite-Difference Time-Domain Modeling of Rigid Surfaces," submitted to *J. Acoust. Soc. Am.*

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- [3] K. J. Shlager and J. B. Schneider, "A 2-D Dispersion Analysis of the W-MRTD Method Using CDF Biorthogonal Wavelets," IEEE AP-S International Symposium and URSI Radio Science Meeting, vol. 3, pp. 244–247, San Antonio, TX, Jun. 2002.
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- [5] M. W. Feise, P. J. Bevelacqua, and J. B. Schneider, "Backward-Wave Meta-Materials for Perfect Lenses," IEEE AP-S International Symposium and URSI Radio Science Meeting, San Antonio, TX, Jun. 2002.
- [6] M. W. Feise, P. J. Bevelacqua, and J. B. Schneider, "Effects of Surface Waves on the Behavior of Perfect Lenses," *Physical Review B*, vol. 66, no. 3, 035113 (five pages), 2002.